Dream, Believe and Achieve!

Simonside Primary School
Pencil and Paper Procedures Policy (Maths)

Background to the Policy

This policy contains the pencil and paper procedures that we shall be teaching at Simonside Primary School. It has been written to encourage consistency and progression throughout the school in order to raise mathematical standards.

Although the focus of the policy is on pencil and paper procedures it is important to recognise that **the ability to calculate mentally lies at the heart of the New Curriculum (Fluency). In addition, these written procedures should be embedded in problem solving and reasoning tasks.**

The **mental methods** specified in the programme of study for teaching mathematics need to be **taught systematically** from Reception onwards and all pupils should be given regular opportunities to develop the necessary skills. **However mental calculation is not at the exclusion of written recording and should be seen as complementary to and not as separate from it.** In every written method there is some element of mental processing. Sharing written methods with the teacher encourages pupils to think about the mental strategies that underpin them and to develop new ideas. Written recording both helps to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

During their time at school pupils should be encouraged to see mathematics as both a spoken and written language. Teachers need to have secure understanding of the stages of development within calculations regardless of whether the method used is a number line, an expanded form of written calculation or a compact method. It is important that pupils do not abandon jottings, empty number line work and mental methods once pencil and paper procedures are introduced. Similarly, practical visual aids and images should be used throughout the school to secure and deepen conceptual understanding. Their use is applicable regardless of age or ability.
Pupils of all ages and abilities should always be encouraged to

- **look at the calculation/problem and the numbers involved.**

  The size or complexity of the numbers involved often influences the chosen method of calculation. Pupils need to have a firm grounding in ‘numbers and the number system’.

  decide upon the best method for them to use.

  The chosen method may be mental calculation with or without jottings, an empty number line, expanded written method, compact written method, calculator or indeed another chosen method that the child is comfortable and accurate in using.

- **complete the calculation and check the appropriateness of their answer.**

  All pupils should be encouraged to estimate the approximate size of a calculation and use this to check the ‘reasonableness’ of their answer.

The long term aim is for pupils to be able to select an efficient method of their choice that is appropriate for a given task. They will do this by always asking themselves

- “Can I do this in my head?”
- “Can I do this by using jottings such as an empty number line?”
- “Do I need to use a pencil and paper procedure?”
- “Do I need a calculator?”
- “Have I estimated my answer?”

Pupils need to be given regular opportunities to use and apply calculation methods efficiently to solve a range of problems.

The following pages of this document contain the written calculation stages of development in addition, subtraction, multiplication and division that we have chosen for our school. The stages act as a guide and there is flexibility within the stages for teachers to use their judgement in order to differentiate for the individual pupil abilities within their class.

Note: flexibility is required when considering children’s ability.

Milestones

The milestones (Chris Quigley) based on the New Curriculum (2014) show the progression in children’s use of written methods of calculation and in the level of complexity.

Complexity	**Methods**
Milestone 1 *** Addition and subtraction:** * Solve one-step problems with addition and subtraction: • Using concrete objects and pictorial representations including those involving numbers, quantities and measures. • Using the addition (+), subtraction (-) and equals (=) signs. • Applying their increasing knowledge of mental and written methods. **Multiplication and division:** * Solve one-step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	**Milestone 1** • **Addition and subtraction:** Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: • One-digit and two-digit numbers to 20, including zero. • A two-digit number and ones. • A two-digit number and tens. • Two two-digit numbers. • Adding three one-digit numbers. • Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot. * **Multiplication and division:** • Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (.), division (÷) and equals (=) signs. • Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.

Complexity	**Methods**
Milestone 2 **Addition and subtraction:** Solve two-step addition and subtraction problems in contexts, deciding which operations and methods to use and why. **Multiplication and division:** Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems (such as n objects are connected to m objects).	**Milestone 2** **Addition and subtraction:** Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate. **Multiplication and division:** Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
Milestone 3 • **Addition and subtraction:** Solve multi-step addition and subtraction problems in contexts, deciding which operations and methods to use and why. * **Multiplication and division:** Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. • Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates. • Use knowledge of the order of operations to carry out calculations involving the four operations.	**Milestone 3** • **Addition and subtraction:** Add and subtract whole numbers with more than 4 digits, including using formal written methods. (columnar addition and subtraction) *** Multiplication and division:** • Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. • Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. • Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.

Written methods for addition of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for addition which they know they can rely on when mental methods are not appropriate. These notes show the stages in building up to using an efficient written method for addition of whole numbers by the end of Year 4.

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

At Simonside Primary School, when counting in ones on a number line we refer to this as steps and when counting in numbers more than one we refer to this as jumps - see stage 1 example. We will also write the number we have stepped or jumped in above the bubble as in all of the examples.

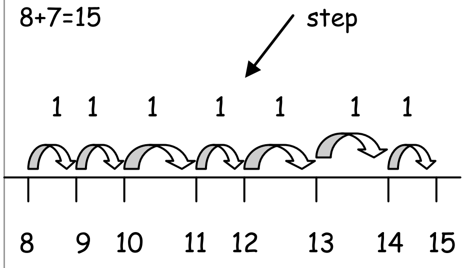
Stage 1: The empty number line

- The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and units separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and units to help them make multiples of ten by adding in steps.
- The empty number line helps to record the steps on the way to calculating the total.

Stage 1

Simonside example

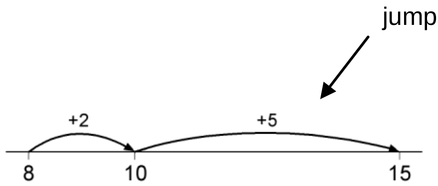
8+7=15



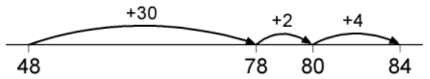
or:

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

8 + 7 = 15



48 + 36 = 84



or:

Stage 2: Partitioning - The next stage is to record mental methods using partitioning. Add the tens and then the units to form partial sums and then add these partial sums. - Partitioning both numbers into tens and units mirrors the column method where units are placed under units and tens under tens. This also links to mental methods.	**Stage 2** Record steps in addition using partitioning: 1). 47 + 76 = 40 + 70 + 7 + 6 = 110 + 13 = 123 2). 47 + 76 = 47 + 70 + 6 = 117 + 6 = 123 Partitioned numbers are then written under one another: 47 + 76 = 40 + 70 = 7 + 6 = 110 + 13 = 123
Stage 3: Expanded method in columns - Move on to a layout showing the addition of the tens to the tens and the units to the units separately. To find the partial sums either the tens or the units can be added first, and the total of the partial sums can be found by adding them in any order. As children gain confidence, ask them to start by adding the units digits first always. - The addition of the tens in the calculation 47 + 76 is described in the words 'forty plus seventy equals one hundred and ten', stressing the link to the related fact 'four plus seven equals eleven'. - The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.	**Stage 3** Write the numbers in columns. **At Simonside, this should be the method used by the end of Year 3.** Adding the units first: 47 + 76 13 110 123 Discuss how adding the units first gives the same answer as adding the tens first. Refine over time to adding the units digits first consistently.
Stage 4: Column method - In this method, recording is reduced further. **Carry digits are recorded below the line,** using the words 'carry ten' or 'carry one hundred', not 'carry one'. - Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.	**Stage 4** **At Simonside, we are aiming for the majority of children to be at this stage by the end of Year 4.** 47 258 366 + 76 + 87 +458 123 345 824 11 11 11 Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.

Written methods for subtraction of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, tens and units in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

At Simonside Primary School, when counting back in ones on a number line we refer to this as steps, and when counting back in numbers more than one we refer to this as jumps - see stage 1 example. We will also write the number we have stepped or jumped back in above the bubble as in all of the examples.

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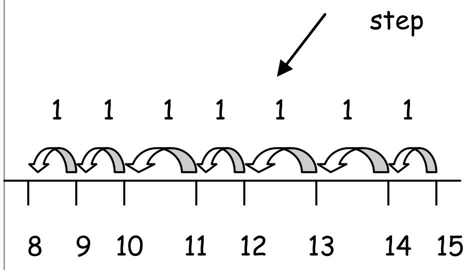
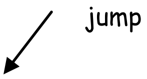
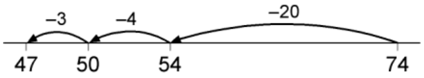
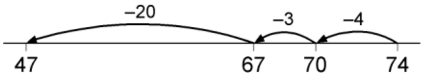
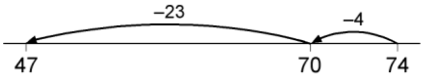
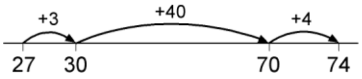
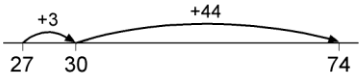
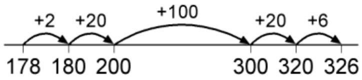
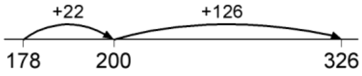
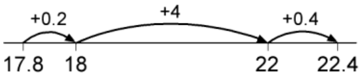
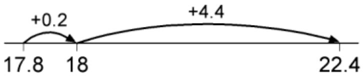
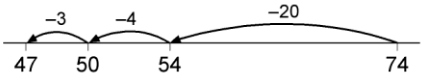
Stage 1: Using the empty number line • The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. • The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. • With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28. The notes below give more detail on the counting-up method using an empty number line.	**Stage 1** Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.  or:  15 – 7 = 8  74 – 27 = 47 worked by counting back:  The steps may be recorded in a different order:  or combined: 
The counting-up method • The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ☐ = 74 mentally.	74 – 27 = 47  or: 74 – 27 = 47 
• With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ☐ = 200 and 200 + ☐ = 326 mentally. • The most compact form of recording remains reasonably efficient.	326 – 178 = 148  or: 326 - 178 = 148 

• The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed. • This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.	22.4 – 17.8 = 4.6  or: 22.4 – 17.8 = 4.6 
Stage 2: Partitioning • Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 – 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.	**Stage 2** Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For example. 1). 74 – 27 = 70 + 4 – 20 – 7 = 60 + 14 – 20 – 7 = 40 + 7 2). 74 – 27 = 74 – 20 – 7 = 54 – 7 = 47 This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line and children may choose to use the number line as their mental jotting. 
Stage 3: Expanded layout, leading to column method • Partitioning the numbers into tens and units and writing one under the other mirrors the column method, where units are placed under units and tens under tens. • This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills. • The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.	**Stage 3** Partitioned numbers are then written under one another: Example: 74 − 27 60 14 70 4 - 20 7 40 7 6 14 7 4 - 2 7 4 7

$$\begin{array}{rrrr} & {\scriptstyle 600} & {\scriptstyle 130} & {\scriptstyle 11} \\ & 700 & 40 & 1 \\ - & 300 & 60 & 7 \\ \hline & 300 & 70 & 4 \\ \hline \end{array}$$

$$\begin{array}{rrrr} & {\scriptstyle 6} & {\scriptstyle 13} & {\scriptstyle 11} \\ & 7 & 4 & 1 \\ - & 3 & 6 & 7 \\ \hline & 3 & 7 & 4 \\ \hline \end{array}$$

Written methods for multiplication of whole numbers

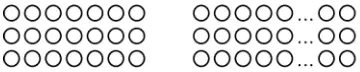
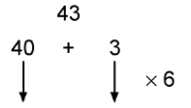
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for multiplication which they know they can rely on when mental methods are not appropriate.

<u>These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.</u>

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

Stage 1: Mental multiplication using partitioning • Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and units to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the units can be multiplied first but it is more common to start with the tens.	**Stage 1** **At Simonside, arrays and repeated addition are used lower down school in key Stage 1.**  3 + 3 + 3 + 3 + 3 + 3 + 3 = 21 Or: 7 × 3 = 21 From Year 3 multiplication facts are written both ways eg: 7 × 3 = 21 or 3 × 7 = 21 **Informal recording in Year 4 might be:**  Also record mental multiplication using partitioning: Note: These methods are based on the distributive law. Children should be introduced to the principle of this law (not its name) in Years 2 and 3, for example when they use their knowledge of the 2, 5 and 10 times-tables to work out multiples of 7:

<table>
<tr>
<td>Stage 2: The grid method
As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.</td>
<td>Stage 2
38 × 7 = 266
$\begin{array}{c|c|c} \times & 30 & 8 \\ \hline 7 & 210 & 56 \end{array} \quad = 266$</td>
</tr>
<tr>
<td>•<u>The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.</u></td>
<td></td>
</tr>
<tr>
<td>Stage 3: Expanded short multiplication
The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is ‘thirty multiplied by seven’, not ‘three times seven’, although the relationship 3 × 7 should be stressed.
<u>Most children should be able to use this expanded method for TU × U by the end of Year 4.</u></td>
<td>Stage 3
$\begin{array}{r} 38 \\ \times \ \ 7 \\ \hline 56 \\ 210 \\ \hline 266 \end{array}$</td>
</tr>
<tr>
<td>Stage 4: Short multiplication
The recording is reduced further, with carry digits recorded below the line.
If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.</td>
<td>Stage 4 - At Simonside we are aiming for children to be at this stage by the end of Year 4.
$\begin{array}{r} 38 \\ \times \underline{\ \ 7} \\ \underline{266} \\ {\scriptstyle 5} \end{array}$
The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.</td>
</tr>
<tr>
<td>Stage 5: Two-digit by two-digit products
Extend to TU × TU, asking children to estimate first.
Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.</td>
<td>Stage 5 – At Simonside we are aiming for children to be at this stage by the end of Year 5.
56 × 27 is approximately 60 × 30 = 1800.
$\begin{array}{c|c|c|c} \times & 20 & 7 & \\ \hline 50 & 1000 & 350 & 1350 \\ \hline 6 & 120 & 42 & 162 \\ \hline & & & 1512 \\ \cline{4-4} & & & {\scriptstyle 1} \end{array}$</td>
</tr>
</table>

<table>
<tr><td>• Reduce the recording, showing the links to the grid method above.</td><td>56 × 27 is approximately 60 × 30 = 1800.
5 6
× 2 7
4 2
3 5 0
1 2 0
10 0 0
15 1 2
1</td></tr>
<tr><td>• Reduce the recording further.
• The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.</td><td>56 × 27 is approximately 60 × 30 = 1800.
5 6
× 2 7
392
1120
1512</td></tr>
<tr><td>Stage 6: Three-digit by two-digit products
• Extend to HTU × TU asking children to estimate first. Start with the grid method.</td><td>Stage 6
286 × 29 is approximately 300 × 30 = 9000.<table><tr><td>×</td><td>200</td><td>80</td><td>6</td></tr><tr><td>20</td><td>4000</td><td>1600</td><td>120</td></tr><tr><td>9</td><td>1800</td><td>720</td><td>54</td></tr></table></td></tr>
<tr><td>• Reduce the recording, showing the links to the grid method above.
• This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.</td><td>2 8 6
× 2 9
5 4
7 2 0
1 8 0 0
1 2 0
1 6 0 0
4 0 0 0
8 2 9 4
2</td></tr>
<tr><td>• Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.</td><td>286 × 29 is approximately 300 × 30 = 9000.
2 8 6
× 2 9</td></tr>
</table>

Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

12÷2 (possible interpretations)

- Equal sharing of 12 between 2
- Finding one half of 12
- Grouping 12 into 2s, which includes
  - -counting forwards in 2s from 0 to 12, or repeatedly adding 2s to reach 12
  - -counting back in 2s from 12 to 0, or repeatedly subtracting 2 from 12

12÷2 (the language of division 1)

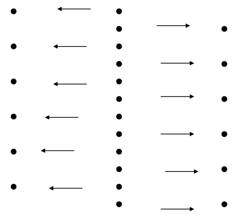
- 12 divided by 2
- 12 divided into 2
- 12 divided between 2

The language of division 2

- 12 divided by 2 associates with the method of grouping or repeated subtraction.
- 12 divided into 2 associates with the method of halving.
- 12 divided between 2 associates with the method of sharing.

Models of division for 12÷2

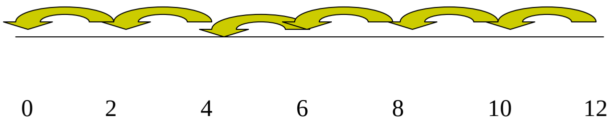
- Sharing between 2 or finding ½ of



Models of division for 12÷2

- Grouping into 2s

- Counting forwards (or backwards in 2s)



NB: group size and step size correspond.

Models of division 12÷2

- Repeatedly subtracting 2 from 12

12-2=

10-2=

8-2=

6-2=4

4-2=2

2-2=0

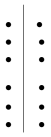
Deriving from knowledge of multiplication facts

6×2=12 → 12÷2=6

Models of division for 12÷2

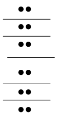
- Sharing or finding ½ of

How many in one column?



Grouping

How many rows are there?



Sharing

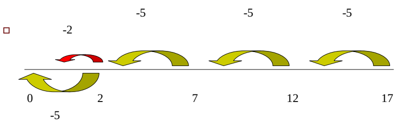
- ☐ Secures understanding of halving and one to one correspondence between objects.
- ☐ Provides little knowledge or skill beyond counting.
- ☐ Provides no image to support understanding of what to do with remainders.
- ☐ As the divisor increases
- ☐ becomes more difficult to visualise
- ☐ becomes less efficient

Grouping

- ☐ Secures understanding that the divisor is important in the calculation
- ☐ Links to the counting in equal steps on a number line
- ☐ Requires sound knowledge of addition and subtraction facts
- ☐ Provides an image to support understanding of what to do with remainders
- ☐ Is more efficient as the divisor increases
- ☐ Provides a firmer basis on which to build pupils' understanding of division

Remainders

- ☐ 17÷5=3 r.2 or 17÷5=3 2/5

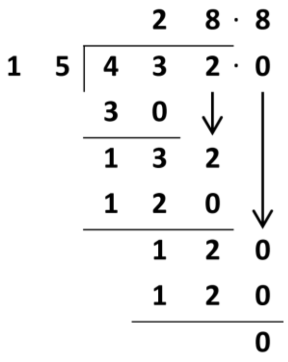


Stage 1: Mental division using partitioning	**Stage 1**
• Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find the total quotient. • Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention. • Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.	One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining ones, then divide each part separately. Informal recording in Year 4 for 84 ÷ 7 might be: $$\begin{tikzcd} & 84 & & \\ 70 \arrow[d] & + & 14 \arrow[d, "\div 7"] & \\ 10 & + & 2 & = 12 \end{tikzcd}$$ At Simonside this method may be used before chunking. In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

Stage 2: Short division of TU ÷ U	**Stage 2**
• ‘Short’ division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning. • Short division of a two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. • For most children this will be at the end of Year 4 or the beginning of Year 5. • The accompanying patter is ‘How many threes divide into 80 so that the answer is a multiple of 10?’ This gives 20 threes or 60, with 20 remaining. We now ask: ‘What is 21divided by three?’ which gives the answer 7.	$\begin{array}{r} 2\ 7 \\ 3\overline{)8\ {}^{2}1} \end{array}$ $-\ \underline{30}\ \ (10)$ 51 $-\ \underline{30}\ \ (10)$ 21 $-\ \underline{21}\ \ (7)$ 0 This is then shortened to: $\begin{array}{r} 2\ 7 \\ 3\overline{)8\ {}^{2}1} \end{array}$ The carry digit ‘2’ represents the 2 tens that have been exchanged for 20 units. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript. The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 units.

Stage 3: ‘Expanded’ method for HTU ÷ U	**Stage 3**
• This method is based on subtracting multiples of the divisor from the number to be divided, the dividend. • For TU ÷ U there is a link to the mental method. • As you record the division, ask: ‘How many nines in 90?’ or ‘What is 90 divided by 9?’ • Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same. • This method, often referred to as ‘chunking’, is based on subtracting multiples of the divisor, or ‘chunks’. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. • Chunking is useful for reminding children of the link between division and repeated subtraction. • However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.	97 ÷ 9 = 10 r. 7 10 r. 7 9) 97 - 90 (10) 7
	196 ÷ 6 = 32 r. 4 32 r. 4 6) 196 - 60 (10) 136 - 60 (10) 76 - 60 (10) 16 - 12 (2) 4 32

• The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that ‘trap’ the HTU dividend. • Estimating has two purposes when doing a division: – to help to choose a starting point for the division; – to check the answer after the calculation. • Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.	To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, … to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40. Start the division by first subtracting 180, leaving 16, and then subtracting the largest possible multiple of 6, which is 12, leaving 4. 32 r. 4 6) 196 - 180 (30) 16 - 12 (2) 4 32 The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.
Stage 4: Short division of HTU ÷ U • ‘Short’ division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction. • The accompanying patter is ‘How many threes in 290?’ (the answer must be a multiple of 10). This gives 90 threes or 270, with 20 remaining. We now ask: ’How many threes in 21?’ which has the answer 7. • Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.	**Stage 4** This is then shortened to: 9 7 3) 2 9 1 Teacher to model that the carry digit ‘2’ represents the 2 tens that have been exchanged for 20 units. In the first recording above it is written in front of the 1 to show that a total of 21 units are to be divided by 3. The 97 written above the line represents the answer: 90 + 7, or 9 tens and 7 units.

Stage 5: Long division The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. The layout on the right, which links to chunking, is in essence the ‘long division’ method. Recording the build-up to the quotient on the left of the calculation keeps the links with ‘chunking’ and reduces the errors that tend to occur with the positioning of the first digit of the quotient. Conventionally the 20, or 2 tens, and the 3 units forming the answer are recorded above the line, as in the second recording.	**Stage 5** How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560. 560 ÷ 24 = 23 r.8 2 3 r. 8 24) 5 6 0 4 8 0 (20) 8 0 - 7 2 (3) 8 23 In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below. 2 3 r. 8 24) 5 6 0 - 4 8 0 8 0 - 7 2 8
Formal Long Division	**Stage 6** 432 ÷ 15 becomes  Answer: 28·8